Introduction to Tensor Numerical Methods in Scientific Computing

Boris N. Khoromskij

Max-Planck-Institute for Mathematics in the Sciences,
Inselstr. 22-26, D-04103 Leipzig, Germany; $bokh$@mis.mpg.de

Abstract

This course presents, in 4 lectures, an introduction to the modern tensor-structured numerical methods in scientific computing. Being developed in the recent years, the tensor numerical methods provide a powerful tool for the efficient computations in higher dimensions overcoming the so-called "curse of dimensionality" (see surveys [1, 2, 3]).

In these lectures I try to display a triple of probably most important ingredients of the tensor approach:

$\diamond$ Analytical methods of separable approximation to multivariate functions and operators in $\mathbb{R}^d$, $d \geq 3$.
$\diamond$ Algebraic low-rank approximation/representation to discretized multi-dimensional functions/operators in the basic tensor formats substituated on the respective multilinear algebra in $\mathbb{R}^{n\times n\times \ldots \times n}$.
$\diamond$ Tensor truncated iterative methods in the Tucker, tensor train (TT) and quantics-TT formats proved to be efficient in applications to the solution of multidimensional equations in electronic structure calculations, quantum molecular dynamics, kinetics models of dilute polymers, stochastic PDEs, to name only a few.

This presentation is based on the contribution by the author's group at MPI MIS, Leipzig as well as on the results obtained in GERRUSLAB collaboration [4]. Detailed literature will be provided in the course of lectures.

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http://personal-homepages.mis.mpg.de/bokh

References

[1] B.N. Khoromskij. *Introduction to Tensor Numerical Methods in Scientific Computing.* Lecture Notes, University/ETH Zuerich, Preprint 06-2011, Uni. Zuerich 2011, pp. 1-238.
http://www.math.uzh.ch/fileadmin/math/preprints/06_11.pdf

[2] B.N. Khoromskij, *Tensors-structured Numerical Methods in Scientific Computing: Survey on Recent Advances.* Chemometrics and Intellingent Laboratory Systems, DOI: 10.1016/j.chemolab.2011.09.001. Preprint 21/2010, MPI MiS Leipzig 2010.

[3] T.G. Kolda and B.W. Bader, *Tensor Decompositions and Applications. SIAM Review*, **51/3**, 2009 455-500.

[4] GERRUSLAB collaboration with the group of Prof. E. Tyrtyshnikov, INM RUS, Moscow.