Workshop on Multidimensional Conservation Laws and Related Problems

June 16-18, 2017

601 Pao Yue-Kong Library

Institute of Natural Sciences, Shanghai Jiao Tong University

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1 General Information

Introduction

The field of multidimensional conservation laws and related problems is experiencing fast and significant advances over the past years. This workshop is aimed to bring together researchers in the field to present recent advances and explore possible collaborations.

Date

June 16-18, 2017

Venue

601 Pao Yue-Kong Library

Scientific Committee

- Shuxing Chen, Fudan University
- Yachun Li, Shanghai Jiao Tong University
- Weike Wang, Shanghai Jiao Tong University
- Ya-Guang Wang, Shanghai Jiao Tong University

Organizers

- Myoungjean Bae, Pohang University of Science and Technology
- Shijin Deng, Shanghai Jiao Tong University
- Beixiang Fang, Shanghai Jiao Tong University
- Qilong Gu, Shanghai Jiao Tong University
- Chunjing Xie, Shanghai Jiao Tong University
- Feng Xie, Shanghai Jiao Tong University
- Xiongfeng Yang, Shanghai Jiao Tong University

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2017-06-16

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11:15 - 12:00	Moon-Jin Kang	Contraction of shocks for the barotropic Navier-Stokes equations
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2017-06-17

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2017-06-18

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11:15 - 12:00	Aifang Qu	Global unbounded weak solution of the Chaplygin gas
14:00 - 14:45	Mikhail Feldman	Uniqueness for shock reflection problem
14:45 - 15:30	Wancheng Sheng	The Two-dimensional Riemann problems for the Euler equations and some related problems of compressible Euler equations for gas dynamics
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16:45 - 17:30	Mingjie Li	Low Mach Number Limit of Multidimensional Steady Flows on the Air Foil Problem

3 Abstract

3.1 2017-06-16

Opening Remarks

Ya-Guang Wang, Shanghai Jiao Tong University
08:20 - 08:30

Free Boundaries and Subsonic Jets

Zhouping Xin, The Chinese University of Hong Kong
08:30 - 09:15

In this talk, I will discuss some issues concerning free boundary value problems associated with steady compressible subsonic jets forming from finite nozzle with given ambient pressure. Such a phenomena has important applications in sciences. Yet the mathematical formulation and analysis of such problems are quite subtle. I will present some recent progress on this problem for a two-dimensional finitely long convergent nozzle with straight solid wall. For such a nozzle, the jet problem can be formulated as follows: for the given ambient pressure and an incoming mass flux, one looks for a subsonic jet flow in a suitable space with the given incoming mass flux such that the flow velocity is in the normal direction at the inlet, the flow satisfies no-flow condition in the nozzle walls, and the flow reaches the ambient pressure at the free boundary.In general, the free boundary consist two distinct parts: one part is the partical path connecting the wall of the nozzle, and other part is a level set of the velocity potential. Such a free boundary value problem does not have a variational structure and there is no background flow to purturb. We

will show that there is an optimal interval such that the jet problem has a unique solution when the length of the nozzle belongs to this interval and there is no such a solution otherwise. This talk is based joint works with ChunpengWang. This research is partially supported by Hong Kong RGC Earmarked Research grants CUHK-14305315 and CUHK-4048/13P, NSFC/RGC Joint Research Scheme N-CUHK443/14, and Focused Innovations Scheme from The Chinese University of Hong Kong.

Stability analysis and quasi-neutral limit for the Euler-Poisson equations

Masahiro Suzuki, Nagoya Institute of Technology
09:15 - 10:00

The purpose of this talk is to mathematically investigate the formation of a plasma sheath near the surface of materials immersed in a bulk plasma, and to obtain qualitative information of such a plasma sheath layer. Specifically, we study the asymptotic behavior and quasi-neutral limit of solutions to the Euler-Poisson equations in a half space or three-dimensional annular domain.

Stability of wave patterns to multi-dimensional viscous conservation laws

Yi Wang, Institute of Applied Mathematics
10:30 - 11:15

In the talk I will first show a L^2-contraction (a L^2-type stability) of large viscous shock waves for the multi-dimensional scalar viscous conservation laws, up to a suitable shift. The shift function, depending both on the time and space variables, solves a viscous Hamilton-Jacobi equation with source terms. We consider a suitably small L^2-perturbation around a viscous planar shock wave with arbitrarily large strength, while the BV-norm or the L^∞-norm of the perturbation can be large. Quite different from the previous results, we do not impose any conditions on the anti-derivative variables of the perturbation around the shock profile. Then I will show the time-asymptotic stability of rarefaction waves to the two-dimensional isentropic compressible Navier-Stokes equations, which gives a first result about the stability the planar rarefaction wave in the multi-dimensional viscous fluids.

Contraction of shocks for the barotropic Navier-Stokes equations

Moon-Jin Kang, KIAS(Korea Institute for Advanced Study)
11:15 - 12:00

We discuss about the contraction of any weak perturbations of viscous shocks for the barotropic Navier-Stokes system. No smallness condition on the perturbations, which do not depend on the viscousity coefficient. Therefore this allows to handle the inviscid limit problem, that is, entropic discontinuous shocks for the isentropic Euler system are unique, and stable in the class of any weak inviscid limit of solutions to the Navier-Stokes system.

Smooth Transonic Flows of Meyer Type in De Laval Nozzles

Chunpeng Wang, Jilin University
14:00 - 14:45

A smooth transonic flow problem is formulated in this talk as follows: For a two-dimensional de Laval nozzle, one looks for a unique smooth transonic steady potential flow of Meyer type whose sonic points are all exceptional and its velocity is along the normal direction at the inlet. If such a flow exists, its sonic curve must be located at the throat of the nozzle and the nozzle should be suitably flat at its throat. We obtain the existence and the uniqueness of such smooth transonic flows in a class of suitably flat de Laval nozzles. For the subsonic-sonic flow, the governing equation is a quasilinear degenerate elliptic equation with the sonic curve as a characteristic degenerate boundary. As to the sonic-supersonic flow, the governing equation is a quasilinear non-strictly hyperbolic equation with strong degeneracy at the sonic curve in the sense that all characteristics from the sonic points coincide with the sonic curve and never approach the supersonic region. The precise asymptotic behavior near the sonic state of the smooth transonic flow is shown and its acceleration is Lipschitz continuous in the de Laval nozzle.

Local Structural Stability of A Multidimensional Complete Rarefaction Wave for the 3-D Steady Supersonic Flow Around a Bend

Gang Xu, Department of mathematics, Jiangsu University
14:45 - 15:30

As indicated in Section 111 of Courant and Friedrichs's book [Supersonic Flow and Shock Waves, Interscience, New York, 1948], a supersonic flow around a bend, one of the most important elementary flows, is effected by a rarefaction wave. This talk is concerned with the local structural stability of a multidimensional complete rarefaction wave for the 3D steady supersonic flow past a large bend. More concretely, by taking the involved analysis on the resulting degenerate hyperbolic equation near the vacuum boundary and establishing the related energy estimates with suitable degenerate weights, we prove the local existence and stability of a multidimensional complete expansion wave including a vacuum boundary when a perturbed supersonic incoming flow moves around the large bend. This is a jointed work with Prof. Yin Huicheng and Dr. Zhu Lu.

Half Space Problems for Euler Equations with Damping

Shijin Deng, Shanghai Jiao Tong University
16:00 - 16:45

In this talk, I will talk about the half space problem for Euler equations with damping in 1-D and 3-D. We restudy the fundamental solution for the Cauchy problem to obtain an exponentially sharp pointwise structure and a clear decomposition of the singular-regular components. Later, both Green's function for initial boundar value problem and fundamental solutions for Cauchy problems are investigated in the transformed domain after Laplace tranform. The symbols are obtained and a connection between Green' s function and fundamental solutions are established for the pointwise space-time structure of Green' s function. Finally, the sharp estimates for

Green' s function together with a priori estimates from the energy method for high order derivatives result in the nonlinear stability of the solution and also the decaying rates.

Stability of steady solutions for the Euler-Poisson system

Ben Duan, School of Mathematics, Dalian University of Technology
16:45 - 17:30

In this talk, we will discuss the Euler-Poisson flows in nozzles. Our motivation and four types of flow patterns will be introduced, including subsonic flows, supersonic flows, transonic shocks and smooth transonic flows. We may focus on the case of subsonic flows, the unique existence of multi-dimensional irrotational flow and 2-dimensional Euler-Poisson flow will be reported.

3.2 2017-06-17

A decomposition result for vector fields in $\mathbb{R}^d$

Stefano Bianchini, SISSA
08:30 - 09:15

The existence and stability of solutions to the ODE

$$\frac{d}{dt}X = (t, X), \qquad X(0, y) = y \tag{1}$$

is classically approached at a pointwise level: for which vector fields there exists a unique solution for *every* $y \in \mathbb{R}$.

In many PDEs systems, however, one has to study the Eulerian formulation

$$\rho_t + {}_x (\rho) = 0, \qquad \rho(0, x) = \rho_0(x), \tag{2}$$

where the relation with the above (Lagrangian) formulation is given by the formula

$$\int \rho(t, x)\psi(t, x)dx = \int \psi(t, X(t, y))\rho_0(y)dy \tag{3}$$

for every test function ψ. This second formulation is in some sense weaker, since one needs only the uniqueness for trajectories which are used by $\rho(t, x)$, in particular for almost all initial points y.

We will review how the synergy between the two formulations allows to generalize the classical ODE theory to more general vector fields with low regularity, for example vector fields_ whose derivative is a measure.

Transport equation with nonlocal velocity

Hantaek Bae, Ulsan National Institute of Science and Technology
09:15 - 10:00

We consider 1D equations with nonlocal velocity field

$$\theta_t + u\theta_x - \delta u_x\theta + \Lambda^\gamma\theta = 0 \tag{4}$$

where $u = \mathcal{N}(\theta)$ is given by one of the form

1. $u = \mathcal{H}\theta$; 2. $u = (1 - \partial_{xx})^{-\alpha}\theta$.

In this talk, we address the existence of weak solutions of (4). When $0 < \gamma < 1$, we take initial data having finite energy. When $\gamma = 1$, we take initial data having infinite energy involving Muckenhoupt weights.

Oscillatory traveling wave solutions to an attractive chemotaxis system

Tong Li, University of Iowa
10:30 - 11:15

We investigate global existence and long time behavior of solutions for PDE models of chemotaxis. In particular, we study oscillatory traveling wave solutions to an attractive chemotaxis system. The convective part of this system is of mixed-type. The oscillatory nature of the traveling wave comes from the fact that one far-field state is in the elliptic region and another in the hyperbolic region. Such traveling wave solutions are shown to be linearly unstable.

Some Results on 3D Prandtl Equations

Tianwen Luo, Tsinghua University
11:15 - 12:00

We obtain a class of weak solutions of the three-dimensional Prandtl equations, which are related to the secondary flows in the three-dimensional boundary layers. This is a joint work with Prof. Zhouping Xin.

Expansion of a compressible gas in vacuum

Denis Serre, Ecole Normale Superieure de Lyon
14:00 - 14:45

Tai-Ping Liu [2] introduced the notion of "physical solution" of the isentropic Euler system when the gas is surrounded by vacuum. This notion can be interpreted by saying that the front is driven by a force resulting from a Hölder singularity of the sound speed. We address the question of when this acceleration appears or when the front just move at constant velocity (ballistic motion).

We know from [1,4] that smooth isentropic flows with a ballistic front exist globally in time, for suitable initial data. In even space dimension, these solutions may persist for all $t \in \mathbb{R}$; we say that they are *eternal*. We derive a sufficient condition in terms of the initial data, under which the boundary singularity must appear. As a consequence, we show that, in contrast to the even-dimensional case, eternal flows with a ballistic front don't exist in odd space dimension. Our argument is related to that of Milnor [3] in his proof of the hairy ball Theorem.

In one space dimension, we give a refined definition of physical solutions. We show that for a shock-free flow, their asymptotics as both ends $t \to \pm\infty$ are intimately related to each other.

See my paper [5]

References

[1] M. Grassin. Global smooth solutions to Euler equations for a perfect gas. *Indiana Univ. Math. J.*, **47** (1998), pp 1397–1432,
[2] Tai-Ping Liu. Compressible flow with damping and vacuum. *Japan J. Indust. Appl. Math.*, **13** (1996), pp 25–32
[3] J. Milnor. Analytic proofs of the "hairy ball theorem" and the Brouwer fixed point theorem. *The American Mathematical Monthly*, **85** (1978), pp 521–524
[4] D. Serre. Solutions classiques globales des équations d'Euler pour un fluide parfait compressible. *Annales de l'Institut Fourier* **47** (1997), pp 139–153
[5] D. Serre. Expansion of a compressible gas in vacuum. *Bulletin of the Institute of Mathematics, Academia Sinica, Taiwan*, **10** (2015), pp 695–716.

Diffraction of a shock into an expansion wavefront for the nonlinear wave system

Eun Heui Kim, California State University Long Beach
14:45 - 15:30

We present the existence of the global solution to transonic self-similar nonlinear wave system. The configuration we study is that a planar shock reflects and diffracts as it hits a semi-infinite rigid screen. The diffracted reflected shock meets the diffracted expansion wave, created by the incident shock that does not hit the screen, and changes continuously from a shock into an expansion. The governing equation changes its type and becomes degenerate as the wave changes continuously from a shock to an expansion. Furthermore the governing equation has multiple free boundaries (transonic shocks) and an additional degenerate sonic boundary (the expansion wave). We discuss an analysis to understand the solution structure near at which the shock strength approaches zero and the shock turns continuously into an expansion wavefront, and show the existence of the global solution to this configuration for the nonlinear wave system. We also present intriguing numerical results.

Quasilinear wave equations in exterior domains

Jun Li, Mathematics Department, Nanjing University
16:00 - 16:45

In the theory of compressible aerodynamics, a basic problem is to consider long time stability of motions of compressible gases in 3-D exterior domains. It can be reformulated as quasilinear wave equations in exterior domains with Neumann boundary conditions. In this talk, I will introduce the related backgrounds both in fluid dynamics and in quasilinear wave equations. In addition, when the equations fulfill null form, I will explain the ideas to prove the global well-posedness of this kind of initial-boundary value problem.

Stability of transonic flows past a wedge

Jun Chen, Southern University of Science and Technology
16:45 - 17:30

We will talk about the stability of transonic flows past a 2-D wedge governed by the full Euler equations. Given a piecewise constant transonic flow past a straight wedge, if the incoming flow and the wedge are perturbed, there exists a unique subsonic solution in the downstream

together with a perturbed shock in between. Corner singularity and decay of the subsonic flow at far field are handled through elliptic estimates. The analysis discloses the relation between the shock polar and the regularity and asymptotic behavior of the downstream subsonic flow.

3.3 2017-06-18

Nonlinear Asymptotic Stability of Lane-Emden Solutions of Viscous Gaseous Stars

Tao Luo, Department of Math,City Univeristy of Hong Kong
08:30 - 09:15

In this talk, I will presnt the nonlinear asyptotic stability of Lane-Emden solutions for viscous gaseous stars in the frame work of free boundary problem with physical vacuum singularity.

Nonlinear instability in Compressible fluids under gravity

Ronghua Pan, Georgia Institute of Technology
09:15 - 10:00

It is known in physics that steady state of compressible fluids under the influence of uniform gravity is stable if and only if the convection is absent. For non-isentropic flow, this stability criterion turns into the monotonicity of entropy in the direction of gravity. In this research, we will show that the instability side with mathematical rigor. This is based on a joint work with Xulong Qin, and Zhen-an Yao.

Necessary and sufficient condition for the existence of global smooth solutions for non-homogeneous multi-dimensioanal conservation law and relating problems

Xiaozhou Yang, Wuhan Institute of Physics and Mathematics, Chinese Academy of Sciences
10:30 - 11:15

In this talk, we will report the necessary and sufficient condition for existence of global smooth solution of the inhomogeneous high dimensional scalar conservation law. We will compare this necessary and sufficient conditions with that of the homogeneous multi-dimensional scalar conservationlaws. In addition, we will give the expressions of higher dimensional shock wave and higher dimensional rarefaction wave of non-homogeneous high dimensional scalar conservation law. In addition, some new structures and new phenomena of multi-dimensional shock wave will be reported.

Global unbounded weak solution of the Chaplygin gas

Aifang Qu, Department of Mathematics, Shanghai Normal University
11:15 - 12:00

Consider the existence of L^1_{loc} solution of the Cauchy problem for the Euler system with state of Chaplygin type. The study of L^1 solution differs from that of the L^∞ solution for this kind of linearly degenerate system significantly. In this talk, we give some discussion on it.

Uniqueness for shock reflection problem

Mikhail Feldman, University of Wisconsin-Madison
14:00 - 14:45

We discuss shock reflection problem for compressible gas dynamics, von Neumann conjectures on transition between regular and Mach reflections, and existence of regular reflection solutions for potential flow equation. Then we will talk about recent results on uniqueness of regular reflection solutions for potential flow equation in a natural class of self-similar solutions. The approach is to reduce the shock reflection problem to a free boundary problem for a nonlinear elliptic equation, and prove uniqueness by a version of method of continuity. A property of solutions important for the proof of uniqueness is convexity of the free boundary. This talk is based on jont work with G.-Q. Chen and W. Xiang.

The Two-dimensional Riemann problems for the Euler equations and some related problems of compressible Euler equations for gas dynamics

Wancheng Sheng, Shanghai University
14:45 - 15:30

In this work, we survey the works on the two-dimensional Riemann problems and some related problems of compressible Euler equations for gas dynamics. It contains four sections: 1. Physical background. 2. Two dimensional Riemann problem of Euler equations. 3. Some numerical results and theoretical results. 4. Some related problems.

Existence of subsonic flows past a body or in nozzles

Chao Chen, College of Mathematics and Computer Science
16:00 - 16:45

In this talk, we will discuss subsonic flows past a body or in nozzles. The existence of subsonic flows with large vorticities past a symmetric body is studied. We also present some ideas on potential flows in a nozzle with two outlets.

Low Mach Number Limit of Multidimensional Steady Flows on the Air Foil Problem

Mingjie Li, Minzu University of China
16:45 - 17:30

In this talk, the low Mach number limit of the steady irrotational Euler flows on the airfoil problem is considered. The limit is on the Hölder space, which means the better uniform estimates. And the convergence rate is ε^2, which is higher than the Klainerman-Majda's result in 1981, due to the irrotational property. This is a joint work with professors Tian-yi Wang and Wei Xiang.