交作业时间: 2018年12月2日，星期一

1. Please expand the following functions on $[-\pi, \pi]$ into Fourier series
   (a) $f(x) = \mathrm{sgn} x$;
   (b) $f(x) = |\cos x|$;
   (c) $f(x) = \begin{cases} ax, & x \in [-\pi, 0), \\ bx, & x \in [0, \pi). \end{cases}$
2. Suppose that the Fourier coefficients of 2π-periodic smooth function f on $[-\pi, \pi]$ are c_n, compute the Fourier coefficients $\tilde{c}_n$ for the following functions
   (a) $g(x) = f(-x)$;
   (b) $F(x) = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) f(x-t) dt$.
3. Suppose that ψ is continuous and monotonic on $[0, +\infty)$ and $\lim_{x \to +\infty} \psi(x) = 0$. Prove
$$\lim_{p \to +\infty} \int_0^{+\infty} \psi(x) \sin(px) dx = 0.$$
4. Compute the the Fourier series and its coefficients for a function $f(x)$ which is defined on $[a, a + 2\pi]$.
5. Please expand the following functions into Fourier series in the associated intervals.
   (a) $f(x) = x$, $x \in [0, 1]$.
   (b) $f(x) = \begin{cases} e^{3x}, & x \in [-1, 0) \\ 0, & x \in [0, 1). \end{cases}$
6. Suppose that $f(x)$ is Riemann integrable or absolutely integrable on $[-\pi, \pi]$. Prove that
   (a) if $f(x) = f(x + \pi)$ for any $x \in [-\pi, \pi]$, then $c_{2n-1} = 0$;
   (b) if $f(x) = -f(x + \pi)$ for any $x \in [-\pi, \pi]$, then $c_{2n} = 0$;
7. Suppose that $f(x)$ is Riemann integrable or absolutely integrable on $[0, \pi/2)$. How to extend f to be a function $\tilde{f}$ on $[-\pi, \pi]$ such that the Fourier series of $\tilde{f}$ has the following form
   (a) $\tilde{f} \sim \sum_{n=1}^{\infty} a_n \cos(2n-1)x$;
   (b) $\tilde{f} \sim \sum_{n=1}^{\infty} b_n \sin 2nx$;