交作业时间: 2018年12月9日，星期一

1. Suppose that $\psi(x)$ is continuous and monotone on $[0,+\infty)$, $\lim_{x\to+\infty}\psi(x)=0$. Prove
$$\lim_{\lambda\to+\infty}\int_0^{+\infty}\psi(x)\sin\lambda x dx=0.$$

2. Suppose that $\psi(x)$ is monotone on $[-\delta,\delta]$. Prove
$$\lim_{\lambda\to+\infty}\int_{-\delta}^{\delta}\left\{\psi(x)-\frac{1}{2}(\psi(0+)+\psi(0-))\right\}\frac{\sin\lambda x}{x}dx=0.$$

3. Show that $\sum_{n=2}^{\infty}\frac{\sin nx}{\ln n}$ and $\sum_{n=2}^{\infty}\frac{\sin nx}{\ln\ln n}$ converge pointwisely, but they cannot be the Fourier series of any Riemann integrable or absolutely integrable functions.

4. Compute the Fourier series for the function
$$f(x)=\begin{cases}0, & x\in[-1,0),\\ x^2, & x\in[0,1).\end{cases}$$
Using this Fourier series to compute the following series
   (a) $\sum_{n=1}^{\infty}\frac{1}{n^2}$.
   (b) $1-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\cdots$.
   (c) $1+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\cdots$.

5. Let
$$f(x)=\begin{cases}\pi-x, & 0<x\le\pi,\\ 0, & x=0,\\ -\pi-x, & -\pi<x<0,\end{cases}$$
   (a) Compute the Fourier series of $f$;
   (b) Does the Fourier series of $f$ converge to $f$ on $[-\pi,\pi]$? Does it converge uniformly?

6. Suppose that $f$ is Riemann integrable or absolutely integrable on $[-\pi,\pi]$. Prove that for any $\varepsilon>0$, there exists a trigonometric polynomial $P_n(x)=\sum_{k=0}^{n}(A_k\cos kx+B_k\sin kx)$ such that
$$\int_{-\pi}^{\pi}|f(x)-P_n(x)|dx<\varepsilon.$$

7. Let $f$ be a $2\pi$-periodic function which is Riemann integrable or absolutely integrable on $(0,2\pi)$. If, in addition, $f$ is monotone decreasing on $(0,2\pi)$, then
$$\int_0^{2\pi}f(x)\sin nx dx\ge 0,\ \ n=1,2,\cdots.$$