交作业时间: 2018年12月16日，星期一

1. page 198: 12, 13
2. page 199: 14
3. page 200: 17
4. Using the Fourier series for the function

$$f(x) = \begin{cases} 1, & x \in [-\pi, 0), \\ 0, & x \in (0, \pi) \end{cases}$$

   to prove that

$$\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}.$$

5. Prove that

$$x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos nx, \quad x \in (-\pi, \pi)$$

   and compute $\sum_{n=1}^{\infty} \frac{1}{n^4}$.

6. Suppose that $f(x)$ is twice continuous differentiable function defined on $(-\infty, +\infty,$ which is also 2π-periodic. Denote

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx dx \quad \text{and} \quad b_n'' = \frac{1}{\pi}\int_{-\pi}^{\pi} f''(x)\sin nx dx.$$

   Prove that if $\sum_{n=1}^{\infty} b_n''$ is absolutely convergent, then

$$\sum_{n=1}^{\infty} \sqrt{|b_n|} < \frac{1}{2}\left(2 + \sum_{n=1}^{\infty} |b_n''|\right).$$

7. Suppose that $f(x)$ is a 2π-periodic continuous function on $(-\infty, +\infty)$. Prove that if the Fourier coefficients of f are always zero, then $f(x) \equiv 0$.
8. Let $f; [0, \pi] \to \mathbb{R}$ be continuous and piecewise differentiable. Suppose that f' is Riemann integrable or absolutely square integrable. If either $f(0) = f(\pi) = 0$ or $\int_0^{\pi} f(x)dx = 0$, then

$$\int_0^{\pi} f^2(x)dx \le \int_0^{\pi} (f')^2(x)dx.$$

   Furthermore, the equality holds if and only if $f(x) = a\cos x$.