Functional Analysis **Homework 4** **Spring 2019**

Due: Monday, April 22, 2019

1. Let H be a Hilbert space and A and B are linear maps from H to H. Suppose that A and B satisfy
$$\langle x, Ay\rangle = \langle x, By\rangle \quad \text{for all } x, y \in H.$$
Prove that $A = B$. If H is a complex Hilbert space and A and B satisfy
$$\langle x, Ax\rangle = \langle x, Bx\rangle \quad \text{for all } x \in H,$$
then prove that $A = B$. What can you say about A and B for real Hilbert spaces?

2. Let $f \in C_0^\infty((0,1))$. Using the Lax-Milgram theorem to prove that the problem
$$\begin{cases} -v'' + \dfrac{1}{10}v' + v = f, \\ v(0) = 0, \quad v(1) = 0 \end{cases} \tag{1}$$
has a unique solution $v \in H_0^1((0,1))$.

3. Exercises 1, 2 on page 53

4. Exercise 3 on page 54

5. Exercise 4 on page 57

6. Exercise 5 on page 58

7. Exercise 6 on page 60

8. Exercises 7, 8, 9 on page 61

9. Exercise 10 on page 62

10. Exercise 2 on page 65