# Homework 2

March 7, 2017, due Mar 13, 2017

**Problem 1.** Prove Theorem 1.7 at page 31 of Braess book.

**Problem 2.** Justify that $u(x,y) = \log\log\frac{2}{r} \in H^1(D)$, where $D$ is the unit disk; and $u(x) = r^{-\alpha} \in H^1$, $\alpha < (n-2)/2$ for $n \geq 3$.

**Problem 3.** Problem 1.14 at page 33 of Braess book.

**Problem 4.** Problem 2.16 at page 43 of Braess book.

**Problem 5.** Consider the boundary value problem

$$-\Delta u = f(x), \quad x \in \Omega. \tag{1}$$

with boundary condition $u(x) = 0$ for $x = (x_1, x_2) \in \partial\Omega$. Let $\Omega = [0,1] \times [0,1]$, we can impose the exact solution as

$$u(x) = x_1(1-x_1)\sin(\pi x_2) + x_2(1-x_2)\sin(\pi x_1). \tag{2}$$

and use $f(x) = -\Delta u = (\pi^2 x_1(1-x_1)+2)\sin(\pi x_2) + (\pi^2 x_2(1-x_2)+2)\sin(\pi x_1)$.

(a) Solve this problem with 5-point finite difference method for $m = 4, 8, 16, 32, 64$. You can use natural ordering for the unknowns. Evaluate the error with respect to the exact solution $u$ in terms of the norm

$$\|v_h\|^2 := h^2 \sum_{i=1}^{m}\sum_{j=1}^{m} v_h^2(ih, jh) \tag{3}$$

where $h = 1/(m+1)$. Justify the convergence rate for 5-point finite difference method. For Julia user, you can refer to `http://nbviewer.jupyter.org/url/homepages.warwick.ac.uk/staff/C.Ortner/julia/Laplacian.ipynb`.