Homework 4

March 22, 2017

Problem 1. Braess book, Chapter 2, 4.4

Problem 2. Braess book, Chapter 2, 4.5

Problem 3. Braess book, Chapter 2, 5.15

Problem 4. Given 1-d finite element mesh in Figure 4, where $h_i = x_i - x_{i-1}$. Formulate

Figure 1: 1d FEM mesh

the Galerkin finite element method for

$$\begin{cases} -u'' = f, & x \in (0,1) \\ u = 0, & x = 0,1 \end{cases} \tag{1}$$

Problem 5. Solve the two-point boundary value problem

$$-u'' + u = f(x), \qquad 0 < x < 1, \qquad u(0) = u(1) = 0, \tag{2}$$

(a) Using the finite element method with piecewise linear basis. Evaluate the local(element) stiffness matrix K_j for the interval $[x_{j-1}, x_j]$, and local(element) mass matrix M_j. The (global) mass matrix is defined as the matrix M, such that $M_{ij} := \int_0^1 \phi_i \phi_j \mathrm{dx}$.

(b) Use those results to construct the global stiffness matrix K and mass matrix M, for $f(x) = x$ evalue the load vector F. Write down the Galerkin form of equation (2). Solve the resulting linear system of the form $(K + M)U = F$.

(c) For $f(x) = x$, the exact solution is $u(x) = x - \dfrac{\sinh x}{\sinh 1}$. Justify the convergence rate for linear basis.