Homework 8

April 19, 2017

Problem 1. Braess book 8.6

Problem 2. Braess book 8.7

Problem 3. Braess book 8.8

Problem 4. Consider the boundary value problem,

$$-\Delta u = f(x), \quad x \in \Omega. \tag{1}$$

with boundary condition $u(x) = 0$ for $x = (x_1, x_2) \in \partial\Omega$. Let $\Omega = [0,1] \times [0,1]$, we can impose the exact solution as

$$u(x) = x_1(1 - x_1)\sin(\pi x_2) + x_2(1 - x_2)\sin(\pi x_1). \tag{2}$$

and use $f(x) = -\Delta u = (\pi^2 x_1(1 - x_1) + 2)\sin(\pi x_2) + (\pi^2 x_2(1 - x_2) + 2)\sin(\pi x_1)$.

(a) Solve this problem with piecewise linear finite element method for $m = 4, 8, 16, 32, 64$, the triangulation is the same as in Figure 4. Compute the entries $a(\phi_1, \phi_i)$, $i = 2, 3, \cdots, 7$, which can be used to construct the global stiffness matrix. ϕ_i is the piecewise linear nodal basis at node x_i. The difference with 5-point finite difference method is that you need to evaluate $\int f\phi_i \, \mathrm{d}x$ for each i instead using the value of f directly, this can be done with an appropriate Gauss-Legendre quadrature rule for triangles which is shown in class.

The Gauss-Legendre quadrature rule can also help evaluate the H^1 error and the L^2 error. Justify the convergence rate we learned in class.

For Julia user, you can refer to `http://nbviewer.jupyter.org/url/homepages.warwick.ac.uk/staff/C.Ortner/julia/FiniteElementMethod.ipynb`.

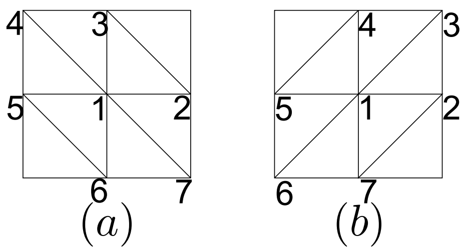

Figure 1: 2d triangulation