Consider the function:
\[ f(x) = \frac{1}{1 + 25x^2}. \]

a) Construct the Lagrange interpolation \( P_n(x) \) of degree \( \leq n \) for \( f(x) \) at equidistant points \( x_i \) between \(-1\) and \(1\) such that:
\[ x_i = \frac{2i}{n} - 1, \quad i \in \{0, 1, \cdots, n\}. \]

Write down and plot \( P_5, P_9 \) on the interval \([-1, 1]\).

b) Explain why when \( n \) increases, the interpolation becomes worse.

c) Construct the Lagrange interpolation at the Chebyshev nodes
\[ x_i = \cos \left( \frac{2i - 1}{2n} \pi \right), \quad i = 1, \cdots, n \]

Write down and plot \( P_5, P_9 \) on the interval \([-1, 1]\). Try to explain why the maximum error in approximating \( f(x) \) is guaranteed to diminish with increasing polynomial degree.

d) Construct the cubic spline interpolation at the equidistant points. Show that when the number of polynomial pieces increase, the interpolation error decreases.

Please print your code and results.

References