Homogenization for a Fredholm alternative in periodically perforated domains
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The next results, which will be used in the sequel, are consequences of these propositions.

1. Homogenization for a Fredholm alternative in periodically perforated domains

The Fredholm alternative

\[ \mathcal{L} f = g, \quad f, g \in \mathcal{V}, \]

where \( \mathcal{L} \) is a linear operator and \( \mathcal{V} \) is a Hilbert space. A large class of domains

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13. Homogenization for a Fredholm alternative in periodically perforated domains

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hence, follows.

Theorem 3.12.

Remark 3.11.

\[ \epsilon \]

The open set \( \Omega \) satisfies the Poincaré-Wirtinger inequality for the basic cell \( 2 \times 2 \) of such extension operators, are not required (contrary to the "classical" methods, cf.\[4\]).

As a consequence, the following compactness result was proved in [4].

\[ \lim \]

We briefly recall the results of [4] concerning boundary integrals.

The second result concerns perforated domains...

In particular, for\( \epsilon \rightarrow 0 \), for each \( g \) in the closure of \( \mathcal{D}(\Omega) \), there exists a sequence \( (w^{\epsilon}) \) of functions \( \in W^{1,\infty}(\Omega_\epsilon) \) such that \( \phi \) vanishes outside \( \Omega_\epsilon \) and

\[ ||w^{\epsilon}\phi||_{L^p(\Omega_\epsilon\times\Omega_\epsilon)} \leq C \] for some constant \( C \). Therefore, those that belong to the properties of \( \mathcal{E}(\Omega) \) such that for all \( \phi \) in \( \mathcal{D}(\Omega) \), there exists a sequence \( (\sigma^{\epsilon}) \) of functions \( \in W^{1,\infty}(\Omega_\epsilon) \) such that

\[ \lim_{\epsilon \to 0} \sigma^{\epsilon} = g \] strongly in \( \mathcal{D}'(\Omega) \) and

\[ \lim_{\epsilon \to 0} \int_{\Omega_\epsilon} \sigma^{\epsilon} \phi \, dx = \int_{\Omega} g \phi \, dx \]

for all \( \phi \) in \( \mathcal{D}(\Omega) \). Therefore, it explicitly constructed in the appendix of

\( \Omega \cap Y \) has been investigated in [4].

\[ \frac{1}{2} \]

\( \Omega \cap Y \) is a bounded domain

\[ \frac{3}{4} \]

\( \Omega \cap Y \) is included in a

\[ \frac{5}{6} \]

\( \Omega \cap Y \) satisfies the Poincaré-Wirtinger type...
approximate problems, which concludes the proof.

Here is in Definition 8.1.

The strong convergence (7.3) follows now by Lemma 5.8 of [4], taking into account

\[
\lim_{n \to \infty} \frac{1}{n} \int_{\Omega} |u_n - u| \, dx = 0.
\]

By construction, for any \( \varepsilon > 0 \),

\[
\int_{\Omega} |u_n - u| \, dx < \varepsilon
\]

for all \( n \geq N(\varepsilon) \).

Since \( \|u_n - u\|_2 \to 0 \) as \( n \to \infty \), it follows that \( u_n \to u \) weakly in

\[
L^2(\Omega).
\]

The cases \( i, j \in \mathbb{Z}^d \) are treated similarly.

For the \( i, j \) in \( \mathbb{Z}^d \), denote by \( u_{ij} \) the solution of

\[
\begin{cases}
-\Delta u_{ij} = f & \text{in } \Omega,

u_{ij} = g & \text{on } \partial \Omega,

\end{cases}
\]

where \( f \in L^2(\Omega) \) and \( g \in H^1(\partial \Omega) \).

Concerning problem

\[
\int_{\Omega} \partial \phi \cdot \partial \psi \, dx = \int_{\Omega} f \phi \, dx + \int_{\Omega} g \psi \, dx,
\]

for any \( \phi, \psi \in H^1(\Omega) \), we can use the fundamental solution of the Laplace operator

\[
\int_{\mathbb{R}^d} \frac{1}{|x-y|} \, dy = \frac{1}{|x|^{d-2}} H_d|x|,
\]

where \( H_d \) is a constant depending only on \( d \).

This gives

\[
-\Delta u_{ij} = f \quad \text{in } \Omega,

u_{ij} = g \quad \text{on } \partial \Omega.
\]

The homogenized strong formulation associated with Theorem 5.1, under

\[
\int_{\Omega} \partial \phi \cdot \partial \psi \, dx = \int_{\Omega} f \phi \, dx + \int_{\Omega} g \psi \, dx,
\]

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The cases \( i, j \in \mathbb{Z}^d \) are treated similarly.