Workshop Series on Advances on Scientific and Engineering Computing (IV) – Theory and Application of Finite Element Method

December 12-13, 2020
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1 General Information

Introduction

Scientific and engineering computing plays a vital role in modern industrial and applied mathematics. Good mathematical models and numerical algorithms can nicely, elegantly or efficiently solve problems which are otherwise difficult and challenging.

The series of SJTU-INS workshops on scientific and engineering aim to bring together leading experts and excellent young researchers inside China to share state of the art research advances in the field of scientific and engineering computing, foster further research collaborations and promote novel and revolutionary research ideas.

The specific theme of this workshop is on theory and application of finite element method.

Acknowledgement

感謝国家自然科学基金、科学与工程计算教育部重点实验室、国防基础科研核科学挑战计划专题和深海重载作业装备集成攻关大平台等项目对本系列会议的支持。

Date

December 12-13, 2020

Registration

No registration fee. Please Register Online.

Venue

Room 306, No. 5 Science Building, Minhang Campus, Shanghai Jiao Tong University

How to arrive: https://ins.sjtu.edu.cn/contact-us

Organizing Committee

- Jianguo Huang, Shanghai Jiao Tong University
- Shi Jin, Shanghai Jiao Tong University
- Wenjun Ying, Shanghai Jiao Tong University

Workshop Series

Workshop on Advances on Scientific and Engineering Computing (I)
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Workshop Series on Advances on Scientific and Engineering Computing (III)
## 2 Schedule

### 2.1 Day 1, 12 December, Saturday

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<td>09:00 - 09:30</td>
<td>Yanping Chen</td>
<td>Adaptive Hybridizable Discontinuous Galerkin Method for Convection Diffusion Equations with Application to Optimal Control</td>
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<td>09:30 - 10:00</td>
<td>Jun Hu</td>
<td>A Family of Mixed Finite Elements for The Biharmonic Equations on Triangular and Tetrahedral Grids</td>
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<td>10:50 - 11:20</td>
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<td>Discontinuous Galerkin Method for A Distributed Optimal Control Problem of Time Fractional Diffusion Equation</td>
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<td>Wei Gong</td>
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<td>Haijun Wu</td>
<td>A Pure Source Transfer Domain Decomposition Method for Helmholtz Equations in Unbounded Domain</td>
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3 Abstracts

3.1 Day 1, 12 December, Saturday

Adaptive Hybridizable Discontinuous Galerkin Method for Convection Diffusion Equations with Application to Optimal Control

Yanping Chen, South China Normal University
09:00 - 09:30

We investigate a posteriori error analysis for hybridizable discontinuous Galerkin methods for convection diffusion equations. A residual-type error estimator without any postprocessing solutions is proposed, and we prove that the error estimator is robust with respect to the error measured by an easily calculated energy norm in the sense that the efficiency and reliability constants are independent of the diffusion coefficient. Then the result and the method, that have been obtained and used for convection diffusion equations, are applied to linear-quadratic distributed optimal control problems governed by convection diffusion equations. We use the variational discretization concept for the approximation of control, and an efficient and reliable error estimator is obtained for the error measured by an energy norm. Note that the estimator is also robust in the sense that the ratio of the upper and lower bounds is independent of the diffusion coefficient. Finally, several examples are provided to validate the performance of the obtained error estimators.

A Family of Mixed Finite Elements for The Biharmonic Equations on Triangular and Tetrahedral Grids

Jun Hu, Peking University
09:30 - 10:00

This talk introduces a new family of mixed finite elements for solving a mixed formulation of the biharmonic equations in two and three dimensions. The symmetric stress is sought in the space H(divdiv) simultaneously with the displacement in L2. Stemming from the structure of H(div) conforming elements for the linear elasticity problems proposed by Hu and Zhang, the H(divdiv) conforming finite element spaces are constructed by imposing the normal continuity of divergence on the H(div) conforming spaces of piecewise symmetric matrix valued polynomials of degree not more than k. The inheritance makes the basis functions easy to compute. The discrete spaces for the displacement are composed of piecewise polynomials of degree not more than k-2 without requiring any continuity. Such mixed finite elements are inf-sup stable on both triangular and tetrahedral grids for polynomial degree not less than 3, and the optimal order of convergence is achieved.

Local Multilevel Methods for Adaptive Discontinuous Finite Elements

Xuejun Xu, Tongji University
10:20 - 10:50

In this talk, I shall present some local multilevel algorithms for solving the linear algebraic systems arising from the adaptive discontinuous finite element methods. The abstract Schwarz
theory is applied to analyze the multilevel methods with Jacobi or Gauss-Seidel smoother performed on local nodes on each level. It is shown that the local multilevel methods are optimal, which means that the convergence rates are independent of the mesh sizes and mesh levels. Numerical results shall be given to confirm our theoretical findings.

Discontinuous Galerkin Method for A Distributed Optimal Control Problem of Time Fractional Diffusion Equation

Xiaoping Xie, Sichuan University
10:50 - 11:20

This talk is devoted to the numerical analysis of a control constrained distributed optimal control problem subject to a time fractional diffusion equation with non-smooth initial data. The solutions of the state and co-state are decomposed into singular and regular parts, and some growth estimates are obtained for the singular parts. Following the variational discretization concept, a full discretization is applied to the state and co-state equations by using conforming linear finite element method in space and piecewise constant discontinuous Galerkin method in time. Error estimates are derived by employing the growth estimates. In particular, graded temporal grids are adopted to obtain the first-order temporal accuracy. Finally, numerical experiments are provided to verify the theoretical results.

On discrete shape gradients of boundary type for PDE-constrained shape optimization

Wei Gong, Chinese Academy of Sciences
11:20 - 11:50

Shape gradients have been widely used in numerical shape gradient descent algorithms for shape optimization. The two types of shape gradients, i.e., the distributed one and the boundary type, are equivalent at the continuous level but exhibit different numerical behaviors after finite element discretization. To be more specific, the boundary type shape gradient is more popular in practice due to its concise formulation and convenience to combine with shape optimization algorithms but has lower numerical accuracy. In this talk we introduce a simple yet useful boundary correction for the normal derivatives of the state and adjoint equations, motivated by their continuous variational forms, to increase the accuracy and possible effectiveness of the boundary shape gradient in PDE-constrained shape optimization. We consider particularly the state equation with Dirichlet boundary conditions and provide a preliminary error estimate for the correction. Numerical results show that the corrected boundary type shape gradient has comparable accuracy to that of the distributed one. Moreover, we give a theoretical explanation for the comparable numerical accuracy of the boundary type shape gradient with that of the distributed shape gradient for Neumann boundary value problems.

A Pure Source Transfer Domain Decomposition Method for Helmholtz Equations in Unbounded Domain

Haijun Wu, Nanjing University
14:00 - 14:30
We propose a pure source transfer domain decomposition method (PSTDDM) for solving the truncated perfectly matched layer (PML) approximation in bounded domain of Helmholtz scattering problem. The method is a modification of the STDDM proposed by [Z. Chen and X. Xiang, SIAM J. Numer. Anal., 51 (2013), pp. 2331–2356]. After decomposing the domain into $N$ non-overlapping layers, the STDDM is composed of two series steps of sources transfers and wave expansions, where $N-1$ truncated PML problems on two adjacent layers and $N-2$ truncated half-space PML problems are solved successively. While the PSTDDM consists merely of two parallel source transfer steps in two opposite directions, and in each step $N-1$ truncated PML problems on two adjacent layers are solved successively. One benefit of such a modification is that the truncated PML problems on two adjacent layers can be further solved by the PSTDDM along directions parallel to the layers. And therefore, we obtain a block-wise PSTDDM on the decomposition composed of $N^2$ squares, which reduces the size of subdomain problems and is more suitable for large-scale problems. Convergences of both the layer-wise PSTDDM and the block-wise PSTDDM are proved for the case of constant wave number. Numerical examples are included to show that the PSTDDM gives good approximations to the discrete Helmholtz equations with constant wave numbers and can be used as an efficient preconditioner in the preconditioned GMRES method for solving the discrete Helmholtz equations with constant and heterogeneous wave numbers.

Two Level Discretizations for Some Classes of PDEs

Liuqiang Zhong, South China Normal University
14:30 - 15:00

First, for nonsymmetric or indefinite linear elliptic PDEs, we obtain the first error estimate in $L^2$-norm for the classical two-grid method, then design and analyze an improved two-grid method by adding one more correction on the coarse space to the classical two-grid method.

Secondly, for second order semilinear elliptic PDEs, we design and analyze a new conforming finite element discretization technique based on iterative two-grid methods, we also present the DG methods and discuss the corresponding iterative two-grid algorithm.

At last, we develop several two-grid methods and two-level additive preconditioners for the Nedelec edge finite element approximation of the time-harmonic Maxwell equations.

Conforming VEM-FEM Hessian Complexes in Three Dimensions

Xuehai Huang, Shanghai University of Finance and Economics
15:00 - 15:30

Two conforming VEM-FEM Hessian complexes in three dimensions are constructed in this work. The degrees of polynomials in both complexes are 5 for $H^2$-conforming VEM, 4 for $H(\text{curl}; \mathcal{S})$-conforming VEM and 3 for $H(\text{div}; \mathcal{T})$-conforming FEM. Three polynomial complexes for tensors are present, including Hessian polynomial complex, divdiv polynomial complex and elasticity polynomial complex. The $H(\text{curl}; \mathcal{S})$-conforming VEM is constructed based on the $H^2$-conforming VEM and the $H(\text{div}; \mathcal{T})$-conforming FEM, by using the idea of the direct sum decomposition of polynomial space for symmetric tensors.
Discontinuous Galerkin Methods with Optimal $L^2$ Accuracy for PDEs with High Order Spatial Derivatives

Yan Xu, University of Science and Technology of China
16:00 - 16:30

In this paper, we formulate and analyze discontinuous Galerkin (DG) methods to solve several partial differential equations (PDEs) with high order spatial derivatives, including the heat equation, a third order wave equation, a fourth order equation and the linear Schroedinger equation in one dimension. Following the idea of local DG (LDG) methods, we first rewrite each PDE into its first order form and then apply a general DG formulation. The numerical fluxes are introduced as linear combinations of average values of fluxes, and jumps of the solution as well as the auxiliary variables at cell interfaces. The main focus of the present work is to identify a sub-family of the numerical fluxes by choosing the coefficients in the linear combinations, so the solution and some auxiliary variables of the proposed DG methods are optimally accurate in the $L^2$ norm. In our analysis, one key component is to design some special projection operator(s), tailored for each choice of numerical fluxes in the sub-family, to eliminate those terms at cell interfaces that would otherwise contribute to the sub-optimality of the error estimates. Our theoretical findings are validated by a set of numerical examples.

Virtual Element Method for General Elliptic Hemivariational Inequalities

Fei Wang, Xi’an Jiaotong University
16:30 - 17:00

An abstract framework of the virtual element method is established for solving general elliptic hemivariational inequalities with or without constraint, and a unified a priori error analysis is given for both cases. Then, virtual element methods of arbitrary order are applied to solve three elliptic hemivariational inequalities arising in contact mechanics, and optimal order error estimates are shown with the linear virtual element solutions. Numerical simulation results are reported on several contact problems; in particular, the numerical convergence orders of the lowest order virtual element solutions are shown to be in good agreement with the theoretical predictions. This is a joint work with Bangmin Wu and Weimin Han.

Deep Learning-based Method Coupled with Small Sample Learning for Solving Partial Differential Equations

Ying Li, Shanghai University
17:00 - 17:30

In this talk, we will introduce a deep learning-based general numerical method coupled with small sample learning (SSL) for solving PDEs. To be more specific, we approximate the solution via a deep feedforward neural network, which is trained to satisfy the PDEs with the initial and boundary conditions. Then the proposed method is modeled to solve an optimization problem by minimizing a designed cost function, which involves the residual of the differential equations, the initial/boundary conditions and the residual of a handful of observations. With a few of sample data, the model can be rectified effectively and the predictive accuracy can be improved. The effectiveness of the proposed method is demonstrated by a wide range of benchmark problems in mathematical physics, including the classical Burgers equations,
Schrodinger equations, Buckley-Leverett equation, Navier-Stokes equation, and Carburizing diffusion equations, which are applied in carburizing diffusion problems in material science. And the results validate that the proposed algorithm is effective, flexible and robust without relying on trial solutions.

3.2 Day 2, 13 December, Sunday

Welcome Remarks

Shi Jin, Shanghai Jiao Tong University
08:45 - 09:00

計算半线性椭圆问题多解的一类谱 Galerkin 型搜索延拓法的收敛性分析

Ziqing Xie, Hunan Normal University
09:00 - 09:30

本文提出计算半线性椭圆边值问题多解的一类高效的谱 Galerkin 型搜索延拓法（SGSEM）。该方法基于模型方程相应线性特征值问题的若干特征函数的线性组合构造多解初值，充分利用了传统搜索延拓法构造多解初值方面的优势。同时，采用插值系数 Legendre-Galerkin 谱方法离散模型问题，具有计算成本低、计算精度高的特点，运动 Schauder 不动点定理和其他技巧，本文严格证明了对应于每个特定真解的数值解的存在性以及限制在改真解一个充分小的领域内的数值解的唯一性，证明了其谱收敛性，数值结果验证了算法的可行性与高效性，并展示了不同类型的多解。

Superconvergence Study of The Direct Discontinuous Galerkin Method and Its Variations for Diffusion equations

Xinghui Zhong, Zhejiang University
09:30 - 10:00

We apply Fourier analysis technique to investigate superconvergence properties of the direct discontinuous Galerkin (DDG) method, DDG method with interface correction (DDGIC), symmetric DDG method and nonsymmetric DDG method. We also include the study of interior penalty DG (IPDG) method, due to its close relation to DDG methods. Error estimates are carried out for both P2 and P3 polynomial approximations. By investigating the quantitative errors at the Lobatto points, we show that DDGIC and symmetric DDG methods are superior, in the sense of obtaining (k+2)th superconvergence orders for both P2 and P3 approximations. Superconvergence order of (k+2) is also observed for IPDG method with P3 polynomial approximations. The errors are sensitive to the choice of numerical flux coefficient for even degree P2 approximations, but are not for odd degree P3 approximations. Numerical experiments are carried out at the same time and the numerical errors match well with the analytically estimated errors.
MAC Scheme for Coupled Stokes Darcy Problems on Non-uniform Grids

Hongxing Rui, Shandong University
10:20 - 10:50

The marker and cell (MAC) method, a finite volume or finite difference method based on staggered grids, has been one of the simplest and most effective numerical schemes for solving the flow problems. The superconvergence on uniform grids for Stokes equations has been observed since 1992 but numerical analysis was not obtained completely. In this talk we will present the analysis of superconvergence for both velocity and pressure for the MAC scheme of Stokes problems and coupled Stokes and porous media flow problems. Numerical experiments using the MAC scheme show agreement of the numerical results with theoretical analysis.

典则 Hamilton 系统的一个辛格式

Yushun Wang, Nanjing Normal University
10:50 - 11:20

辛算法因长时间计算稳定，精确的特性而受到广泛关注。一直以来，形式简单的辛格式只有隐中点格式和辛欧拉格式。这两个格式也因此被广泛应用于多个领域。报告给出了第二个、第四个简单形式的辛格式，并进一步给出了一个一般的辛格式，使得前面这些形式简单的辛格式成为该格式的特例。我们从不同角度重构了该格式，证明了格式的辛性质，并基于该格式得到了两类带参数的任意高阶的辛格式。通过调整格式中的参数，我们可以把任意阶精度的辛格式改造成能量守恒格式，从而给出了构造任意阶精度保能量格式的一类新方法。

The Elastodynamic Models and Numerical Computation

Xiaoqin Shen, Xi’an University of Technology
11:20 - 11:50

The theory of elastic shells is one of the important branches of the theory of elasticity. In this talk, we discuss several kinds of elastodynamic shell models, i.e., the time-dependent Koiter model, the time-dependent generalized membrane model, the time-dependent elliptic membrane model and the time-dependent flexural model, which have not been addressed numerically. We propose space-time full discretization schemes for several models. The corresponding analyses of existence, uniqueness, stability, convergence and priori error estimates are given. Finally, we provide numerical experiments with several kinds of shells to demonstrate the efficiency of models and the stability and convergence of numerical schemes.

A BPX-type Preconditioner for Integral Fractional Diffusion on Bounded Domains

Shuonan Wu, Peking University
14:00 - 14:30

We propose and analyze a BPX-type preconditioner for the integral fractional Laplacian on bounded domains. For either quasi-uniform grids or bisection grids, we show that the condition number of the resulting systems remains uniformly bounded with respect to both the number
of levels and the fractional power. This is a joint work with Prof. Juan Pablo Borthagaray (UDELAR), Prof. Ricardo H. Nochetto (UMD), and Prof. Jinchao Xu (PSU).

An Efficient Numerical Method to Compute Ground States of A Rotating Bose-Einstein Condensate with Impurities

Yongjun Yuan, Hunan Normal University
14:30 - 15:00

In this talk, an efficient and accurate numerical method is developed to compute ground states of a novel rotating BEC-impurity system. The key idea is twofold. First, since the impurities are localized in a small box potential, two grids are designed to compute the ground states of the model problem, i.e., a general grid for a suitable large domain to compute the wave function of BEC and an imbedded grid for a small domain determined by the size of a box potential to calculate the wave functions of impurities. Then, a preconditioned conjugate gradient (PCG) method is applied to solve the ground states of the rotating BEC-impurity system. Extensive numerical results of ground states for the BEC-impurity system are reported to show the efficiency of our method and demonstrate interesting physical phenomena.

Convergence of A Second-order Energy-decaying Method for The Viscous Rotating Shallow Water Equation

Jilu Wang, Beijing Computational Science Research Center
15:30 - 16:00

An implicit energy-decaying modified Crank-Nicolson time-stepping method is constructed for the viscous rotating shallow water equation on the plane. Existence, uniqueness and convergence of semidiscrete solutions are proved by using Schaefer’s fixed point theorem and $H2$ estimates of the discretized hyperbolic-parabolic system. For practical computation, the semidiscrete method is further discretized in space, resulting in a fully discrete energy-decaying finite element scheme. A fixed-point iterative method is proposed for solving the nonlinear algebraic system. The numerical results show that the proposed method requires only a few iterations to achieve the desired accuracy, with second-order convergence in time, and preserves energy decay well.

A Level Set Method for Shape and Topology Optimization

Shengfeng Zhu, East China Normal University
16:00 - 16:30

We develop a finite-element based level set method for numerically for shape and topology optimization. By combining the shape sensitivity analysis and level set method, a gradient descent algorithm is proposed to solve the model problems. Different from solving the nonlinear Hamilton–Jacobi equation with finite differences in traditional level set methods, we solve the level-set convection equation and reinitialization equation using the Galerkin finite element method. The methodology can handle shape and topological changes in both regular and irregular design regions. Numerical results are presented.