HW1

1. (From Qingyang Li)

   Suppose we have uniformly spaced grid points $\{x_i\}_{i=0}^n \subset [-4, 4]$ such that $x_0 = -4, x_n = 4$. The function values of $y = e^x$ are given at these points. Define $h = 8/n$. For any given x, we use quadric interpolation to approximate $y = e^x$ on $[-4, 4]$ using suitably chosen three points. Estimate how large n should be if we want the interpolation error to be no more than 10^{-5}.

2. (From Qingyang Li) Let $\ell_{n,j}(x)$ be the Lagrangian interpolation basis functions for grid points $\{x_j\}_{j=0}^n$. Use the uniqueness of interpolation polynomial to show that

   - $$\sum_{j=0}^{n} x_j^k \ell_j(x) \equiv x^k, \ \forall k = 0, 1, \cdots n.$$
   - $$\sum_{j=0}^{n} (x - x_j)^k \ell_j(x) \equiv 0, \ \forall k = 1, \cdots n.$$

3. (From Burden)

   Construct the interpolating polynomial of degree four using the Newton's interpolatory divided-difference algorithm, for the data

   $$f(0) = -6, \ f(0.1) = -5.8948, \ f(0.3) = -5.65, \ f(0.6) = -5.178, \ f(1) = -4.28.$$

   Update the polynomial if we add the datum $f(1.1) = -3.996$ into the table.

   For this problem, attach your code as appendix.

4. Please analyze the complexity of computing the interpolation basis functions if we have $n+1$ points in Lagrange interpolation. For Newton's interpolation, if we add one more point into the list, what is the complexity for the extra work?

5. (From Qingyang Li)

   Let $f(x) = \frac{1}{1+x^2}$, $x \in [-5, 5]$. Consider uniform grid on this interval with $n+1$ points (i.e., we have with $x_0 = -5, x_{10} = 5$ and $x_{j+1} - x_j = \frac{10}{n}$).

- Compute and plot the Lagrange interpolation polynomials on this interval for $n = 10$ and $n = 20$. Explain what you see.
- If we use the piecewise linear function to interpolate, plot the interpolation function again for $n = 10$ and $n = 20$. Find an error bound for $n = 20$.

6. (From Qingyang Li) Given the function values as follows

x	0.25	0.30	0.39	0.45	0.53
y	0.5	0.5477	0.6245	0.6708	0.7280

Write a short code to compute the cubic spline functions on $[0.25, 0.53]$ given the conditions $S''(0.25) = S''(0.53) = 0$. Plot the function $S(x)$.

You should explain how you construct $S(\cdot)$ in your homework and attach the code as appendix.