HW4

1. Find the LU decomposition of the following matrix

$$A = \begin{pmatrix} 2 & -1 & 1 \\ 3 & 3 & 9 \\ 3 & 3 & 5 \end{pmatrix}$$

2. Implement the LU decomposition for a tridiagonal matrix and write a code for how to use the decomposed matrices to solve a tridiagonal linear system. Apply your code to solve the linear system in Hw1 No. 6. (You need to program for this problem.)

3. Find the Cholesy decomposition of the tridiagonal matrix

$$A = \begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{pmatrix}$$

and use your decomposition to solve the linear system

$$Ax = e_1,$$

where e_1 is the vector with the first entry to be 1, and other entries being zero.

4. (a) Compute the $\| \cdot \|_p$ norm ($p = 2, \infty$) for

$$A = \begin{pmatrix} 99 & 98 \\ 98 & 97 \end{pmatrix}.$$

Compute its conditional number for $\| \cdot \|_2$.

(b) Compute using MATLAB the 2-condition number $\kappa(A)_2$ of the Hilbert matrices H_3 and H_5.

5. Consider the linear system

$$\begin{pmatrix} 1 & 2 \\ 1.0001 & 2 \end{pmatrix} x = \begin{pmatrix} 3 \\ 3.0001 \end{pmatrix}.$$

It has a solution $x^T = (1, 1)$.

If we change it to

$$\begin{pmatrix} 1 & 2 \\ 1.00008 & 2 \end{pmatrix} x = \begin{pmatrix} 3 \\ 3.0001 \end{pmatrix}.$$

Compute the solution. What is the relative change of x? Compare your numerical result to the error bound in lecture. Then, conclude whether the linear system is well-conditioned or not.

6. Consider two matrices A, B. Consider the condition number corresponding to some operator norm $\|\cdot\|$. Show that

$$\kappa(AB) \leq \kappa(A)\kappa(B).$$

7. Let A be a matrix with size $n \times n$. Suppose the matrix has rank n. Show that $A^T A$ is positive definite. What is the relation between the 2-condition numbers of $A^T A$ and A?