

# Homework 4, due Apr 1, 2014

April 3, 2014

**Problem 1.** Let  $A$  be the tridiagonal matrix generated by either central difference or piecewise linear finite element in 1D. Namely,  $A$  is a  $d \times d$  Toeplitz matrix, with  $\tau_0 = 2$ ,  $\tau_1 = \tau_{-1} = -1$ ,  $b = \mathbf{1} \in \mathbb{R}^d$ .

Implement the following methods to solve the linear equation  $Ax = b$ ,

- (a) LU factorization,
- (b) Jacobi iteration,
- (c) Gauss-Seidel iteration,
- (d) Steepest decent,
- (e) Conjugate gradient,

For the iterative methods (b)-(e), run each method for 1000 steps, record the error ( $\|Ax^k - b\|$ ) at each step, and plot the error with respect to the number of steps. Explain your observation.

For convergence rate of Jacobi method and Gauss-Seidel method, read Iserles 266-267.

For the algorithm of conjugate gradient method, read Iserles 316-317.

Note: You can use Matlab commmand *toeplitz* to generate the matrix  $A$ ,