Homework 8

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Problem 1. Beam-Warming method (LeVeque book, P212)

1. Find the truncation error of Beam-Warming method.
2. Conduct a von Neumann stability analysis for the Beam-Warming method.

Problem 2. Revisit the Cauchy problem

$$\begin{cases} \dfrac{\partial u}{\partial t} + \dfrac{\partial u}{\partial x} = 0, \quad x \in \mathbb{R}, t \in [0, T] \\ u(x, 0) = u_0(x), \end{cases} \tag{1}$$

where

$$u_0(x) = \begin{cases} 1, & x \leq 0, \\ 0, & x > 0. \end{cases} \tag{2}$$

Take $h = 0.01$, $k/h = 0.5$. Use Upwind method, Lax-Wendroff method, and Beam-Warming method to compute the solution up to $t_n = 0.5$. Compare the numerical solution with the exact solution. Notice that in this case, you just need to choose a sufficient large interval, for example, $[0, 1]$ to compute the numerical solution, and the boundary condition can be chosen as $U(0, t) = 1$, and $U(1, t) = 0$.

- Observe the ossicilations due to dispersion of the L-W and B-W methods, try to explain the observation by calculating the group velocities of the L-W and B-W methods separately (LeVeque book 325-327).

Problem 3. Derive the modifed equation for the Lax-Wendroff method.