Homework 9

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April 30, 2014

Problem 1. Beam-Warming method continue (LeVeque book, P212)

- Use quadratic interpolation on U_{j-2}^n, U_{j-1}^n and U_j^n to construct Beam-Warming method.

Problem 2. For Leap-Frog method, find the condition on h and k required by CFL condition.

Problem 3. Consider the method

$$U_j^{n+1} = U_j^n - \frac{ak}{2h}(U_j^n - U_{j-1}^n + U_j^{n+1} - U_{j-1}^{n+1}). \tag{1}$$

for the advection equation $u_t + au_x = 0$ on $0 \le x \le 1$ with periodic boundary conditions.

1. This method can be viewed as the trapezoidal method applied to an ODE system $U'(t) = AU(t)$ arising from a method of lines discretization of the advection equation. What is the matrix A? Don't forget the boundary conditions.

2. Suppose we want to fix the Courant number ak/h as $k,\ h \to 0$. For what range of Courant numbers will the method be stable if $a > 0$? If $a < 0$? Justify your answers in terms of eigenvalues of the matrix A from part (a) and the stability regions of the trapezoidal method.

3. Apply von Neumann stability analysis to the method (1). What is the amplification factor $g(\xi)$?

4. For what range of ak/h will the CFL condition be satisfied for this method (with periodic boundary conditions)?

5. Suppose we use the same method (1) for the initial-boundary value problem with $u(0,t) = g_0(t)$ specified. Since the method has a one-sided stencil, no numerical boundary condition is needed at the right boundary (the formula (1) can be applied at x_{m+1}). For what range of ak/h will the CFL condition be satisfied in this case? What are the eigenvalues of the A matrix for this case and when will the method be stable?

Problem 4. Email your midterm project report to me by May 8. Late report will get 1 point per day deducted from your score.