Homework 7

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Problem 1. Draw the butterfly diagram for FFT when $N = 8$.

Problem 2. Use the implementation of the Fast Fourier Transform algorithm in `http://www.codeproject.com/Articles/9388/How-to-implement-the-FFT-algorithm` (which also appears in Numerical Recipe `http://apps.nrbook.com/c/index.html`, Chapter 12, page 507-508). Let $N = 64$, $h = 1/64$, and the sample points are $x_j = jh$, $j = 0, 1, \ldots, N-1$. Apply the FFT transform to the sampled data $\{g_j\}_{j=0}^{N-1}$ from the following functions g, where $g_j = g(x_j)$. Plot the Discrete Fourier coefficients with respect to frequency, explain what you obeserved (Note: in general, the Discrete Fourier transform of g_j are complex numbers, you can plot their real part and imaginary part separately, you can also plot their amplitude and phase separately).

- $g(x) = \cos(2\pi x)$ and $g(x) = \cos(128\pi x)$
- $g(x) = x^2$ and $g(x) = x^{10}$.
- $g(x) = \sin(x^2)$ and $g(x) = \sin(100x^2)$

Problem 3. Verify the following perperty for the matrix S used in the Discrete Sine Transform, where $S_{jk} = \sin(\frac{jk\pi}{N+1})$, $j, k = 1, \ldots, N$. Let S_k be the $k-$th column of S,

(a) S_k are orthognal to each other.

(b) $\|S_k\| = \sqrt{\frac{N+1}{2}}$.

(c) $S^{-1} = \frac{2}{N+1}S$.