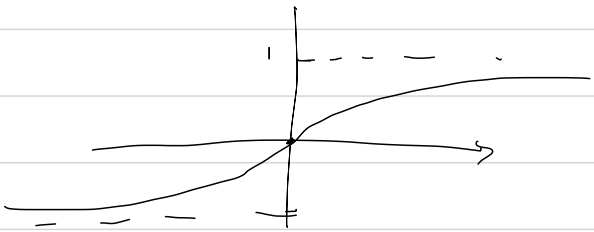
lec3. theory for F-Principle

Activation: $\sigma(x) = \tanh(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$



Two-layer neural network   1d in   1d out

$$\boxed{h(x) = \sum_{j=1}^{m} a_j \sigma(w_j x + b_j) \qquad x \in \mathbb{R}}$$

$$\hat{f}(k) = \int_{-\infty}^{+\infty} f(x) e^{-ikx} \, dx$$

$$\widehat{\sigma(w_j x + b_j)}(k) = \frac{2\pi i}{|w_j|} \exp\left(\frac{i b_j k}{w_j}\right) \frac{1}{\exp\left(-\frac{\pi k}{2 w_j}\right) - \exp\left(\frac{\pi k}{2 w_j}\right)}$$

Assume $\frac{\pi k}{2 w_j} > 0$

$$\approx -\frac{2\pi i}{|w_j|} \exp\left(\frac{i b_j k}{w_j}\right) \exp\left(-\frac{\pi k}{2 w_j}\right)$$

$$\Rightarrow \hat{h}(k) = -\sum_{j=1}^{m} \frac{2\pi a_j i}{|w_j|} \exp\left(\frac{i b_j k}{w_j}\right) \exp\left(-\frac{\pi k}{2 w_j}\right)$$

Define $D(k) = \hat{h}(k) - \hat{f}(k) = A(k) e^{i\phi(k)}$

Define loss at k:

$$\boxed{L(k) = \frac{1}{2} |D(k)|^2}$$

Define loss at frequency domain

$$L = \frac{1}{2} \int |D(k)|^2 \, dk$$

Parseval 等式

$$L = \frac{1}{2}\int \left(f(x) - h(x)\right)^2 dx$$

$$\theta^{t+1} = \theta^{t} - \eta \frac{\partial L}{\partial \theta}$$ GD $D(k)\overline{D(k)}$

$$\frac{\partial L(k)}{\partial a} = \frac{1}{2} D(k) \frac{\partial D(k)}{\partial a} + c.c$$

$$= D(k) \exp\left(-\frac{\pi k}{2w_j}\right) G_1(\theta)$$

$$= A(k) \exp\left(-\frac{\pi k}{2w_j}\right) G(\theta)$$

$$\frac{\partial L(k)}{\partial \theta} = A(k) \exp\left(-\left|\frac{\pi k}{2w_j}\right|\right) G(\theta)$$